Sediment Transport Simulation Model for a River System in Barak Basin Using Muskingum Routing & Rating Equations for Multiple Inflows

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Abstract—In present study, a sediment transport simulation model for the Barak river reach in Meghna Basin has been developed. This model combines Muskingum flow model and Sediment Transport Rating curve for river sections to develop Water discharge-Sediment Concentration Model and Water discharge-Sediment Discharge Model for a river reach. The model is based on the correlation between water discharge and sediment concentration at site and is used to develop integrated sediment and water transport model for a reach. The model in multi input-single equivalent output form is applied to a river system in Barak river basin having water and sediment flow from different upstream sub-basin. Downstream flow values. Model parameters are estimated by Multi Objective optimization using Genetic Algorithm. The model results are evaluated using statistical measure.

Comparison of the computed water and sediment flow graphs with the values observed at the downstream section indicate satisfactory model performances.

1. INTRODUCTION

Sediment transport is caused by process of weathering and erosion in upland areas and river channel. During transport, the abrasion separtes the particle into coarse, medium and fine grained sediments. The sediment causes heavy effect on downstream channel, reservoirs or dams. Several models have been developed and designed as MUSLE,RUSLE, SLEMSA and WEPP for prediction of sediment load based on erosion model. To compute sediment load at downstream section in river system we combine Muskingum Routing model and Sediment Transport Curve as power function symbolising non-linearity of the model with very weak value of coefficient of correlation between sediment concentration and water discharge.

1.1 Formulae: Transport Curve & Application of Muskingum Routing Methods

Sediment Transport curve develops probability correlation between sediment concentration and water discharge of a stream resulting to estimate bed and total load transport with time-series data.

$$Q_{s} = \alpha Q_{w}^{\beta} \tag{1}$$

Qs-Suspended sediment discharge in tons/day;

 Q_w -Water discharge in ft³/sec or m³/sec;

α- the intercept;

 β - The slope;

The Muskingum routing method defines a variable dischargestorage relationship and models storage volume of flooding in a river channel. Storage functions for linear model in flow stream given by Equation (2) below:

S=K[XI+(1-X)Q] ; X=Weighting factor(0 to 0.5) in Muskingum model; K=Proportionality coefficient or Muskingum model parameter having dimension of time. So, change in storage values over time interval 't' & 't+ Δ t' can be expressed as Equation (3) below:

 $S_{t+\Delta t} \text{ - } S_t \text{=} K \left\{ [XI_{t+\Delta t} \text{+} (1\text{-}X)Q_{t+\Delta t}] \text{ - } [XI_t \text{+} (1\text{-}X)Q_t] \right\}$

Change in storage can be also expressed as:

$$\frac{\mathbf{I}_{t}+\mathbf{I}_{t+\Delta t}}{2}.\Delta t - \frac{\mathbf{Q}_{t}+\mathbf{Q}_{t+\Delta t}}{2}.\Delta t$$
(4)

Combing equations (1) & (2) and simplifying,

$$Q_{t+\Delta t} = C_1 I_t + C_2 I_{t+\Delta t} + C_3 Q_t$$
 (5)

The basic Muskingum routing model obtained combining weighted storage & continuity equation as:

$$S_{w,t} = k[xQ_{w,t}^{u} + (1-x)Q_{w,t}^{d}]$$
(6)

$$Q_{w,(t+\Delta t)}^{d} = C_1 Q_{w,t}^{u} + C_2 Q_{w,(t+\Delta t)}^{u} + C_3 Q_{w,t}^{d}$$
(8)

Where,
$$C_1 = \frac{\Delta t + 2KX}{\Delta t + 2K(1-X)}$$
 (8.a)

$$C_2 = \frac{\Delta t - 2KX}{\Delta t + 2K(1 - X)}$$
(8.b)

$$C_3 = \frac{2K(1-X)-\Delta t}{\Delta t+2K(1-X)}$$
(8.c)

$$C_1 + C_2 + C_3 = 1$$
 (9)

U=Upstream section; d=Downstream Section; t=Time instant; w=Water discharge instant; $S_{w,t}$ =Water Storage at time t; C_1 , C_2 & C_3 =Muskingum routing coefficients.

For river system with multiple inflows [1] at a point is:

$$q_{(t+\Delta t)} = C_1 I_t^{e,r} + C_2 I_{(t+\Delta t)}^{e,r} + C_3 q_t$$
(10)

$$I_t^{e,r} = \sum_{p=1}^n \sigma^{p,r} I_t^p \tag{11}$$

 $\sigma^{p,r}$ =Shift factor with transfer of flow from p to r; I_t^p =Flow at point p; q=Outflow at downstream station of river system at time instant; $I_t^{e,r}$ =Equivalent inflow at point r in basin for n flows at different location.

Water discharge & sediment concentration for section is expressed in power form written as [3]

$$C_{s,t}^* = \alpha_* (Q_{w,t}^*)^{\beta_*}$$
 (12)

$$Q_{s,t}^{*} = C_{s,t}^{*} Q_{w,t}^{*} = \alpha_{*} (Q_{w,t}^{*})^{\beta_{*}+1}$$
(13)

'*' indicates a section; $Q_{w,t}^*$ =Instataneous water discharge at time t for a section(vol./time); $C_{s,t}^*$ =Instantaneous sediment concentration at time t (wt./vol); $Q_{s,t}^*$ =Instantaneous sediment discharge at time t(wt./time); $\alpha(*) & \beta(*)$ =rating curve parameters reflecting sediment discharge charecterestics at a section. These represents univocal link between sediment concentration & water discharge.The relationship is single valued representing individual correspondence between dependent & independent variable defining inverse functional relationship for site.Using the relations water discharge can be

written :
$$Q_{w,t}^* = (C_{s,t}^* / \alpha_*)^{1/\beta_*}$$
 (14)

$$Q_{w,t}^{*} = (Q_{s,t}^{*} / \alpha_{*})^{1/(\beta_{*}+1)}$$
(15)

In case of multiple river reach, sediment inflows can be accomplished by aggregating multiple inflows at point in the basin can be written in the form of,

$$C_{s,t}^{u,p} = \sum_{p=1}^{n} C_{s,t}^{u,p} = \sum_{p=1}^{n} \sigma^{u,p} \left(\frac{C_{s,t}^{u}}{\alpha_{u}} \right)^{\frac{1}{\beta_{u}}}$$
(16)

$$Q_{s,t}^{u,p} = \sum_{p=1}^{n} Q_{s,t}^{u,p} = \sum_{p=1}^{n} \sigma^{u,p} \left(\frac{Q_{s,t}^{u}}{\alpha_{u}} \right)^{\frac{1}{(\beta_{u}+1)}}$$
(17)

 $C_{s,t}^{u,p} \& Q_{s,t}^{u,p} =$ Equivalent sediment concentration and sediment discharge from point u to p; $C_{s,t}^{u} \& Q_{s,t}^{u} =$ Sediment concentration and discharge at point u. Rating curve parameters, α_u =Dimension of sediment density & β_u = an exponent. $\sigma^{u,p}$ =Shift factor associated with transfer of flow from u to p. Using water flow estimate obtained by applying Eqs. (14 &15), then combining w.r.t Eqs. (16 & 17) incorporating in Eq. (8), Muskingum model can be in terms of sediment variable for multiple river reaches can be derived or rewritten as:

$$C_{s,(t+\Delta t)}^{d} = \alpha_{d} \left[C_{1} \left[\sum_{p=1}^{n} \sigma^{u,p} \left(\frac{C_{s,t}^{u,p}}{\alpha_{u,p}} \right)^{\frac{1}{\beta_{u,p}}} \right]^{+} (1 - C_{1} - C_{3}) \right]$$

$$\left[\left(\sum_{p=1}^{n} \sigma^{u,p} \left(\frac{C_{s,(t+\Delta t)}^{u,p}}{\alpha_{u,p}} \right)^{\frac{1}{\beta_{u,p}}} \right)^{\frac{1}{\beta_{u,p}}} + C_{3} \left(\frac{C_{s,t}^{d}}{\alpha_{d}} \right)^{\frac{1}{\beta_{d}}} \right]^{\beta_{d}}$$

$$(18)$$

'd'=denotes downstream section; 's'=sediment; $\alpha_{u,p}$ & $\beta_{u,p}$ =denotes upstream sediment rating curve parameters at point p; 'u'= Upstream section; C=Computed value at a position; O=Observed value at a position.

$$Q_{s,(t+\Delta t)}^{d} = \alpha_{d} \left[C_{l} \left(\sum_{p=1}^{n} \sigma^{u,p} \left(\frac{Q_{s,t}^{u,p}}{\alpha_{u,p}} \right)^{\overline{(\beta_{u,p}+1)}} \right)^{+(1-C_{1}-C_{3})} \right]$$

$$\left(\sum_{p=1}^{n} \sigma^{u,p} \left(\frac{Q_{s,(t+\Delta t)}^{u,p}}{\alpha_{u,p}} \right)^{\overline{(\beta_{u,p}+1)}} \right)^{+C_{3}} \left(\frac{Q_{s,t}^{d}}{\alpha_{d}} \right)^{\overline{(\beta_{d}+1)}} \right]^{(\beta_{d}+1)}$$

$$(17)$$

$$\min f = \sum_{i=1}^{n} \left(O_{(t+\Delta t)}^{i} - C_{(t+\Delta t)}^{i} \right)^{2}$$
(19)

To simulate the model the river system is categorized into upstream and downstream network. These formulas represent the water discharge-sediment concentration & sediment discharge model (WSCM & WSDM) and subsequently the values of water discharge model at downstream end. The computation is done by lagging the observed value for a day and minimizing the function between these observed and computed values to relatively satisfy continuity. To calibrate the model the three functions are minimized to estimate the model parameters by multi objective optimization using genetic algorithm in MATLAB.

$$OME = \frac{1}{N} \sum_{i=1}^{n} \left(O_i^{0} - C_i^{c} \right)$$
(20)

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{n} \left(O_i^o - C_i^c\right)^2}$$
(21)

$$CORR = \frac{\left(\sum_{i=1}^{n} \left(O_{i}^{o} - \overline{X}\right)^{*} \left(C_{i}^{c} - \overline{Y}\right)\right)}{\sqrt{\sum_{i=1}^{n} \left(O_{i}^{o} - \overline{X}\right)^{2} * \left(C_{i}^{c} - \overline{Y}\right)^{2}}}$$
(22)

OME= Objective Mean Error.

RMSE= Root Mean Square Error.

CORR= Coefficient of Correlation.

2. DATA COLLECTION & STUDY AREA MAP

The Barak river system falls under jurisdiction of Meghna Basin circle and the data is collected from Central Water Commission, Shillong. The sediment concentration and water discharge data series from 1st June to 30th October-2003, 2004, 2005 is used for the calibration of the model.



Fig. 1: Map of Barak River in Meghna Basin

3. RESULTS

Table 1: Sediment Model for Upstream (U) Network & Downstream (D) Network:

Parameter	WSCM-	WSDM-	WSCM-	WSDM-
	U/S	U/S	D/S	D/S
Κ	8.704	8.704	10.26	10.26
Х	0.0049	0.0049	0.036	0.036
αFulertal	5.25	5.25	2.14	2.14
βFulertal	4.39	4.39	0.87	0.87
σFulertal	0.731	0.731	4.18	4.18
αDholai	4.927	4.927	1.554	1.554
βDholai	4.304	4.304	4.29	4.29
σDholai	0.702	0.702	0.688	0.688
αMatijhuri	N.A		5.766	5.766
βMatijhuri	N.A		4.6	4.6
σMatijhuri	N.A		3.458	3.458
αd	0.624	0.624	0.026	0.026
βd	4.08	4.08	0.21	0.21
Upstream	Fulertal, Dholai		Fulertal, Dholai,	
Network			Matijhuri	
d/s(d)	A.P Ghat		B.P Ghat	
C1	0.0593	0.0593	0.084	0.084
C2	0.0499	0.0499	0.012	0.012
C3	0.8908	0.8908	0.904	0.904
OME	0.00076	57.32	0.0004	15.11
RMSE	0.0276	119.466	0.0088	45.41
CORR	0.994	0.973	0.997	0.986

N.A- Not Applicable to the Network



Fig. 2. Upstream Sediment Concentration (in mg/L)-Y axis vs. Time(in days)-X axis.



Fig. 3: Upstream Sediment Discharge(in kg/sec)-Y axis vs. Time(in days)-x axis.



Fig. 4. Downstream Sediment Concentration (in mg/L)-Y axis vs. Time(in days)-X axis.



Fig. 5: Downstream Sediment Discharge (in kg/sec)-Y axis vs. Time(in days)-X axis.

Fable	2: Water	Discharge Model for Upstream (U)
	Network	& Downstream (D) Network:	

Parameters	Upstream	Downstream
K	1.91	2.486
Х	0.000091	0.000073
σFulertal	0.886	0.965
σDholai	2.6	3.257
σTulagram	0.508	0.000038
σMatijhuri	N.A	1.22

Upstream	Fulertal, Dholai,	Fulertal, Dholai,
Network	Tulagram	Tulagram, Matijhuri
d/s(d)	A.P Ghat	B.P Ghat
C1	0.2076	0.168
C2	0.2074	0.167
C3	0.5849	0.665
OME	32.429	11.045
RMSE	192.86	225.55
CORR	0.983	0.967



Fig. 6: Upstream Water Discharge (in m³/sec)-Y axis vs. Time (in days)-X axis



Fig. 7: Downstream Water Discharge (in m³/sec)-Y axis vs. Time (in days)-X axis

The results observed in the study of the Barak River system has been estimated considering inflows from multiple tributary as Rukni and Sonai joining Barak river in Upstream Network with Annapurna (A.P) Ghat at downstream end. The river network completes at Badarpur (B.P) Ghat as downstream end with Katakhal river joining the upstream rivers. The data period used for calibration is 458 days but few sediment data for Sonai river was not available, so it was not taken into consideration for Sediment model. The model performance is checked by statistical measures in the above table and results proved to be satisfactory. The comparison between the observed and computed value is presented in hydrograph and sediment graph. The computed values closely follow the observed values in above figures.

The effect of sediment discharge at downstream section can also be studied in this model by restricting the flow of tributaries upstream as future perspective. The restriction can be done as one or two tributary at a time but taking note that the flow of main river should not be restricted during this process.

The model is highly non-linear and genetic algorithm can optimize non-linear function effectively providing best set of values for identification of unknown parameters to compute downstream values.

4. CONCLUSION

From calculation and computation done above we can Fig. that Sediment Transport model using Muskingum equations is a better alternative for drawing a relationship between watersediment values for a river system. In this single model we evaluate the value of three function sediment concentration, sediment discharge and water discharge for multiple tributary inflows with single equivalent outflow at a time. It indicates that if a set of values for model parameters are available the model can estimate the three functions at both network irrespective of time of data series. The scope of restricting the flow of tributary also signifies the effect from each tributary. The model is useful for sustaining stability of river banks and estimating impact of sediment accumulation in dams or reservoirs.

A similar type of forecasting and simulation model was successfully tested on Mississippi River basin from Chester to Thebes.

The long range of input data series for simulating the three functions may lack some accuracy with observed data series. The equivalent inflow itself from multiple tributary upstream can have error which is used to compute the downstream values at time interval.

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